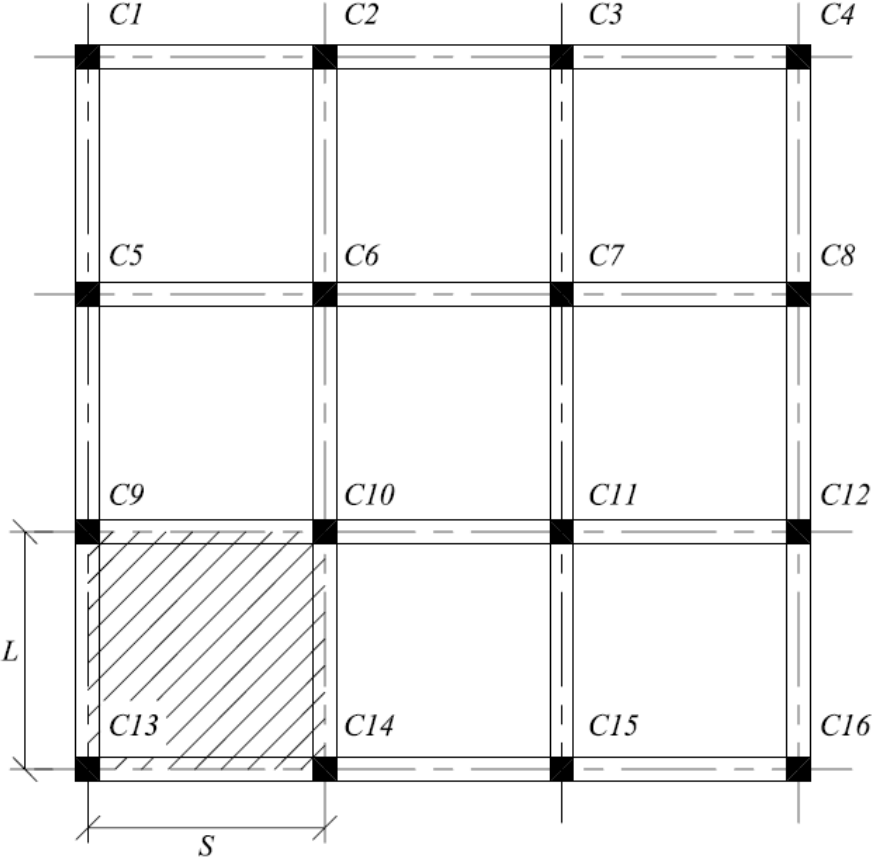


DESIGN OF REINFORCED CONCRETE STRUCTURES II
TWO-WAY SLABS

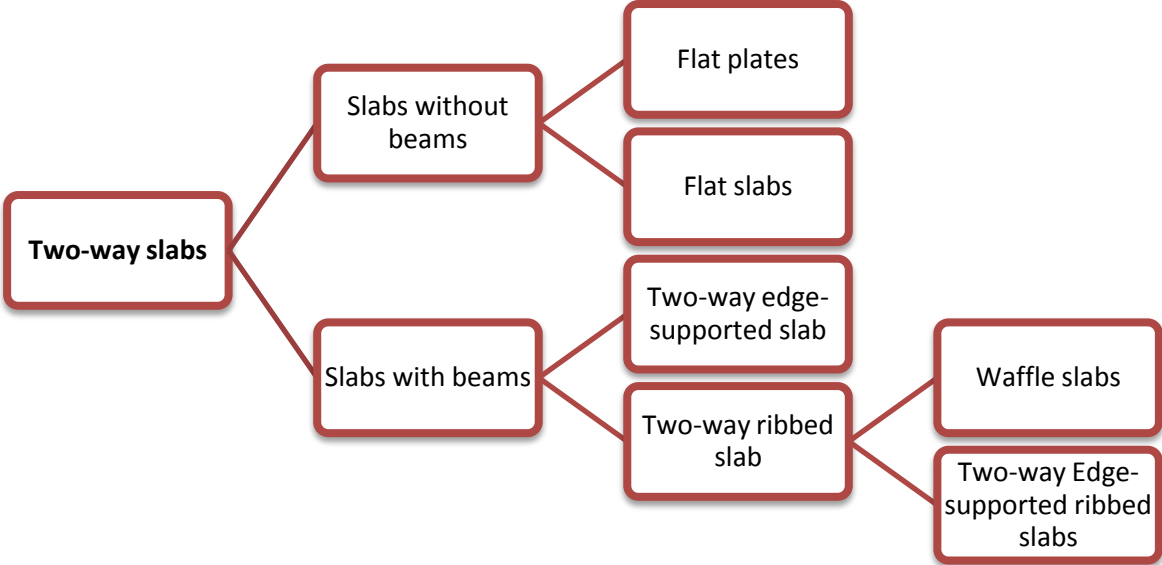
1. Introduction

When the ratio (L/S) is less than 2.0, slab is called two-way slab, as shown in the fig. below. Bending will take place in the two directions in a dish-like form. Accordingly, main reinforcement is required in the two directions.

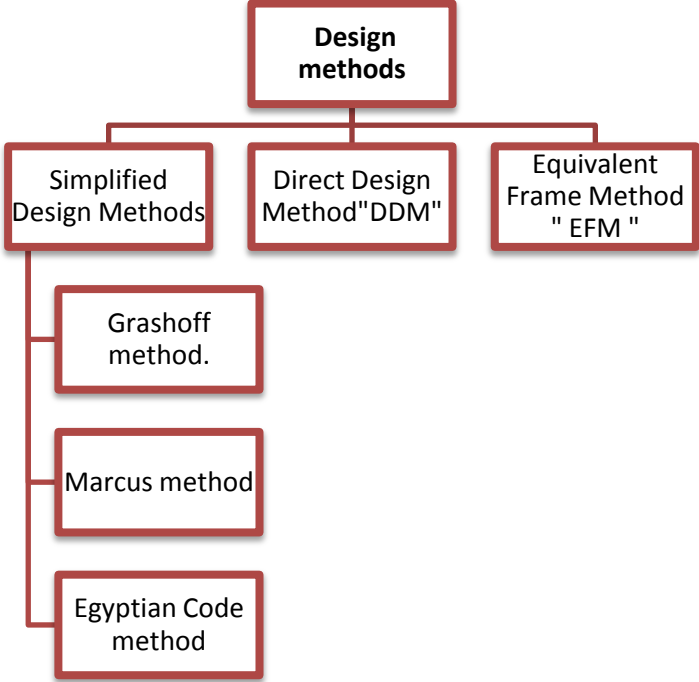


DESIGN OF REINFORCED CONCRETE STRUCTURES II
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2.Types Of Two Way Slabs



3. Design Methods



DESIGN OF REINFORCED CONCRETE STRUCTURES II
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4. Direct Design Method "D.D.M"

Before Discussion Of this Method, we have to study some concepts:

1. Limitations:

1. Three or more spans in each direction.
2. Variation in successive spans $\nless 33\%$ ($\frac{l_2 - l_1}{l_2} \times 100\% \nless 33\%$).
3. LL $\nless 2$ DL
4. Column offset $\nless 10\%$ in each direction.
5. L/B $\nless 2$.
6. For slabs on beams, for one panel $\frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} \nless 0.2$
 $\nless 5.0$.

2. Determination of Two way slab thickness:

- **Case 1** : interior and edge beams are exist.

$$h_{min} = \frac{l_n(0.8 + f_y/14000)}{36 + 5\beta(1 + \beta_s)}$$

$$h_{max} = \frac{l_n(0.8 + f_y/14000)}{36}$$

Where:

l_n : is the largest clear distance in the longest direction of panels.

S_n : is the clear distance in the short direction in the panel.

$$B = \frac{l_n}{S_n}$$

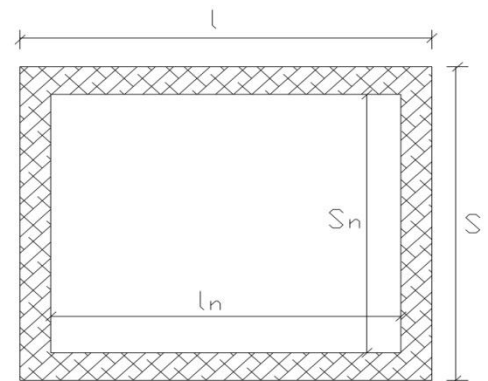
$$B_s = \frac{\text{length of continues edges in the panel}}{\text{Total perimeter of the panel}}$$

Example for finding B_s : for fig. shown:

For panel 1 ... $B_s = \frac{l+s}{2l+2s} = \frac{1}{2}$

For panel 5 ... $B_s = \frac{2l+2s}{2l+2s} = 1$

So **h** to be used should be : $h_{min} < h < h_{max}$



1	2	3
4	5	6
7	8	9

DESIGN OF REINFORCED CONCRETE STRUCTURES II
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- **Case 2:** interior beams are not existing, thickness can be found according to table 8.8, page 339.

Table 8.8: Minimum thickness of slabs without interior beams*

Without drop panels			With drop panels		
Exterior panels		Interior panels	Exterior panels		Interior panels
Without edge beams	With edge beams		Without edge beams	With edge beams	
$l_n / 30$	$l_n / 33$	$l_n / 33$	$l_n / 33$	$l_n / 36$	$l_n / 36$

* $f_y = 4200 \text{ kg/cm}^2$

3. Estimating dimensions of interior and exterior beams sections:

Dimensions can be estimated from the following figures:

Where: b = beam width,
 h = slab thickness,
 a = beam thickness.

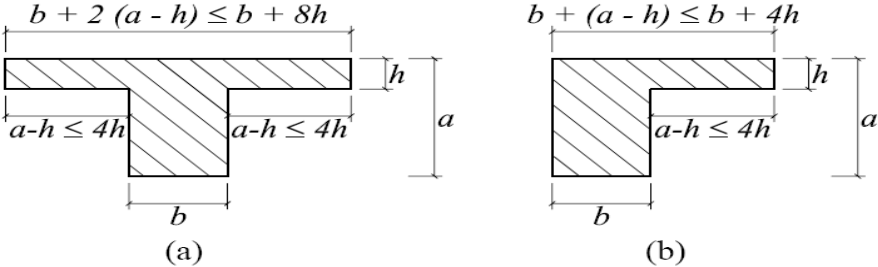
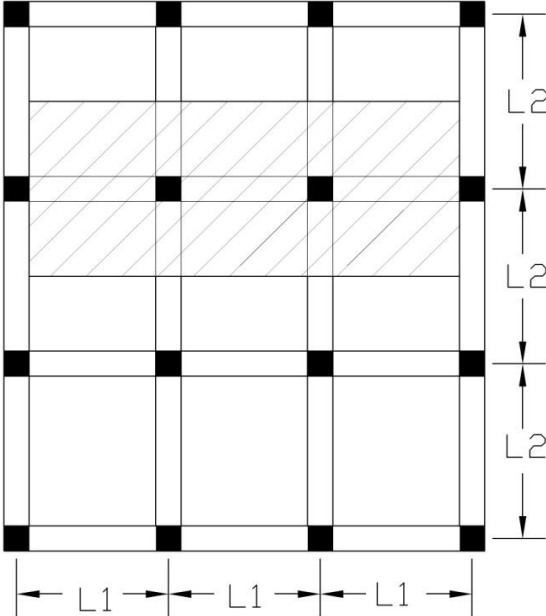


Figure 8.20: Effective beam section; (a) interior beam; (b) exterior beam

DESIGN OF REINFORCED CONCRETE STRUCTURES II
TWO-WAY SLABS

► **Design Procedures**

Discussion will be done to one representative strip in the horizontal and vertical directions; the same procedure can be used for the other strips.



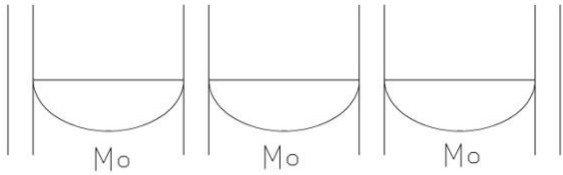
a- **Determination of total factored Static Moment M_o :**

$$M_o = W_u \times \text{Strip width} \times l_n^2 / 8$$

where:

W_u : total factored load in t/m^2 .

l_n = clear distance in the direction of strip, and not less than $0.65l_1$.

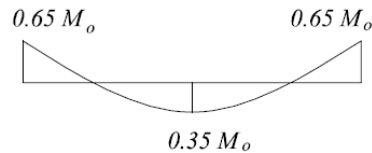


DESIGN OF REINFORCED CONCRETE STRUCTURES II
TWO-WAY SLABS

b- Distribution of the total factored static moment to negative and positive moments:

I. For interior Spans:

According to the code, the moments can be distributed according to factors shown in the figure:

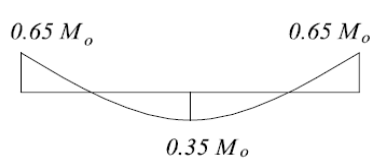


II. For Edge Spans :

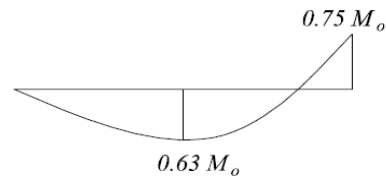
Table 8.9: Distribution of total static moment in end spans

	(1)	(2)	(3)		(4)
	Exterior edge unrestrained	Beams between all supports	No beams between interior supports Without edge beam	No beams between interior supports With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0.00	0.16	0.26	0.30	0.65

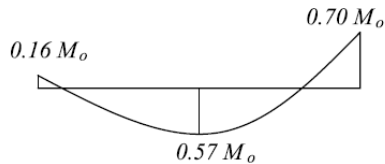
Static Mom. M_o can be distributed, according to factors given in the table 8.9, page 341.



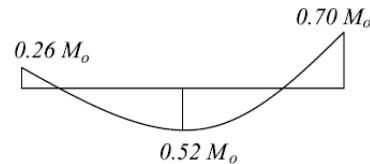
(a) Exterior edge fully restrained



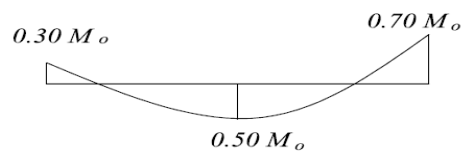
(b) Exterior edge unrestrained



(c) beams between all supports



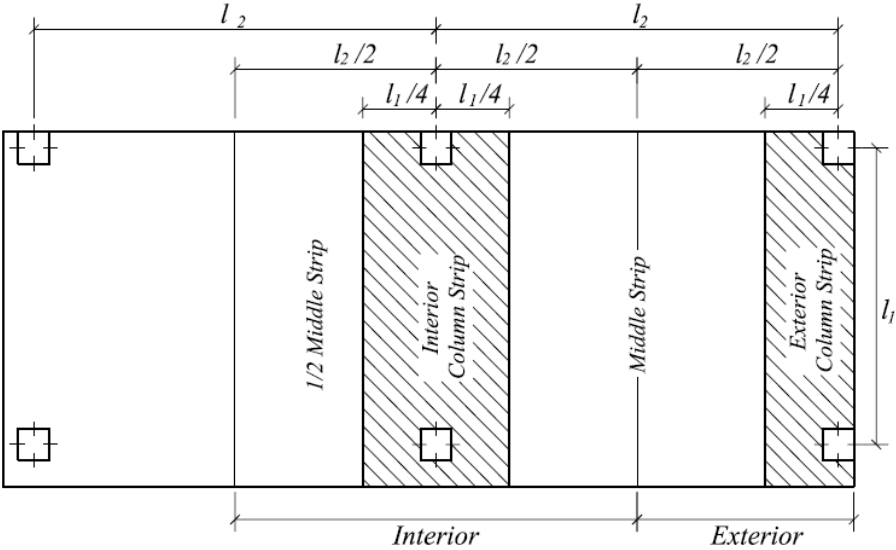
(d) No beams between interior supports and without edge beams



(e) No beams between interior supports and with edge beams

DESIGN OF REINFORCED CONCRETE STRUCTURES II
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c- Distribution of the positive and negative factored moments to the Column and middle strips:



Note: width of column strip is equal to $0.25l_1$ or $0.25l_2$ which is smaller.

l_1 : length in the direction of strip, center to center between columns.

l_2 : length in the direction perpendicular to l_1 .

I. Determination of factored moments on column and middle strips:

- Finding α and β_t :

➤ $\alpha = \frac{I_b}{I_s}$

Note: α is given only for **the beams in the direction** of the strip

α : is ratio of flexural stiffness .

I_b : Moment of inertia of the beam in the direction of strip... can be found from fig.8.14 and fig.8.15, pages 310 and 311.

I_s : Moment of inertia of slab = $\frac{1}{12} \times \text{strip width} \times h^3$, where h is slab thickness.

➤ $\beta_t = \frac{E_{cb}C}{2E_{cs} I_s}$,

Note: β_t is given only for **edge beams perpendicular** to the strip

β_t : Ratio of torsional stiffness

E_{cb} and E_{cs} are the modulus of elasticity of concrete for beam and slab.

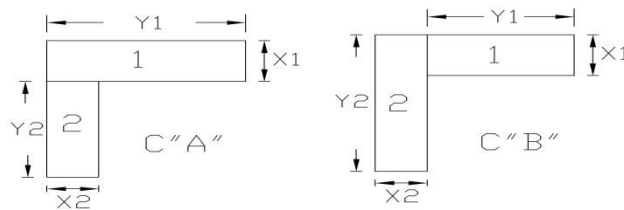
DESIGN OF REINFORCED CONCRETE STRUCTURES II
TWO-WAY SLABS

C: Cross sectional constant defines torsional properties $\Rightarrow C = \sum \left(1 - 0.63 \frac{x}{y} \right) \left(\frac{x^3 y}{3} \right)$

X: smallest dimension in the section of **edge beam**.

Y: Largest dimension in the section of **edge beam**.

Note: the C relation is applicable directly for rectangular section only, but when used for L-Shape beams, we should divide it to two rectangular sections and find C.



C "A" = C₁ + C₂ for A and C "B" = C₁ + C₂ for B.

C to be used = Max (C "A" , C "B").

When α and β_t are found, factors for moment can be found from table 8.10 page 343 for the column strip.

Table 8.10: Column strip factored moments

		l_2 / l_1	0.50	1.0	2.0
Exterior negative factored moment	$\alpha_1 l_2 / l_1 = 0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	75	75	75
	$\alpha_1 l_2 / l_1 \geq 1$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	90	75	45
Positive factored moment	$\alpha_1 l_2 / l_1 = 0$		60	60	60
	$\alpha_1 l_2 / l_1 \geq 1$		90	75	45
Interior negative factored moment	$\alpha_1 l_2 / l_1 = 0$		75	75	75
	$\alpha_1 l_2 / l_1 \geq 1$		90	75	45

Notes:

- $\alpha l_2 / l_1 = 0.0$, when there is no interior beams in the direction of strip under consideration.
- $\beta_t = 0.0$, when there is no exterior "edge" beams perpendicular to the strip under consideration.

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After finding the moments on the column strip, Moments on the middle strip is the remain.

II. For the moment on the beam “ if exist ” :

If: $\alpha l_2/l_1 \geq 1$... The beam moment is 85% of the moment of the column strip.

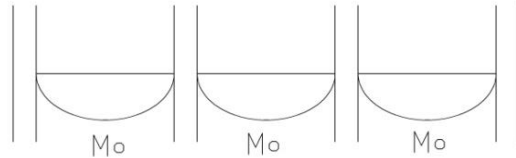
$\alpha l_2/l_1 = 0$... there is no beam .. mom. = 0

$0 < \alpha l_2/l_1 < 1$... Interpolation have to be done between 0 and 85% to find percentage of moment on the beam from that of the column strip.

** The Mom. on the remain part of column strip = Tot. Mom. on the column strip – Mom. on the beam.

Summary:

1- Find M_0 :



2- Distribute M_0 into +ve and -ve Mom.



3- Distribute Mom. Into column strip and Middle Strip.



4- Distribute Mom. In column strip into Mom. On beam and remained slab.



After calculating Moments, we can find the ρ , then A_{st} required

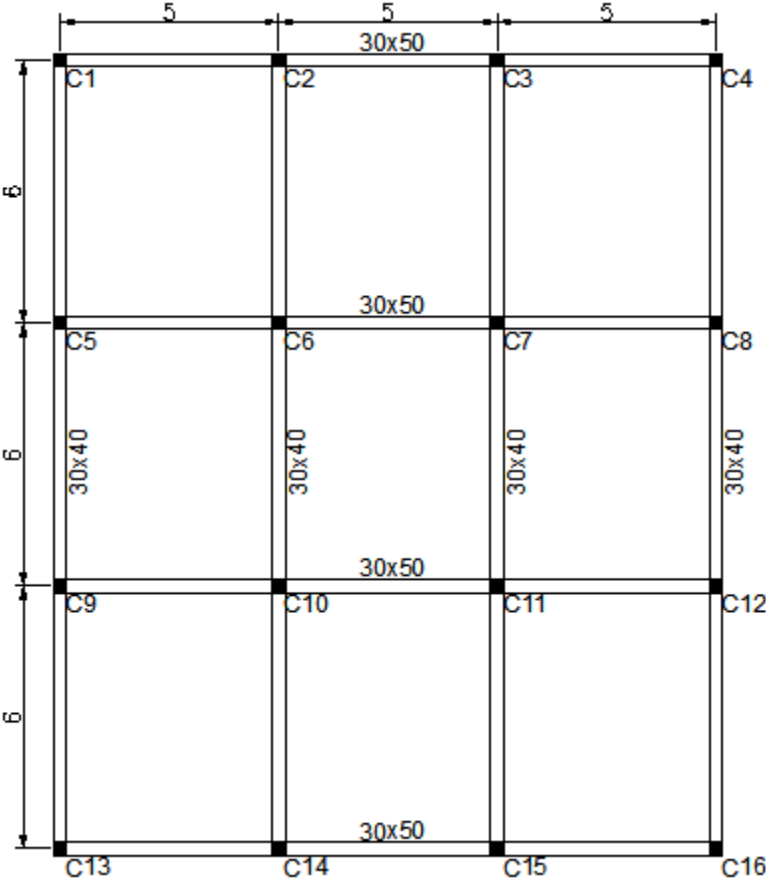
DESIGN OF REINFORCED CONCRETE STRUCTURES II
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Example 1:

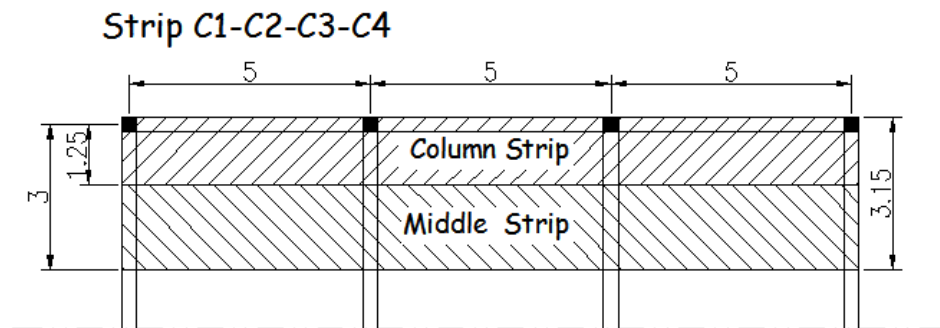
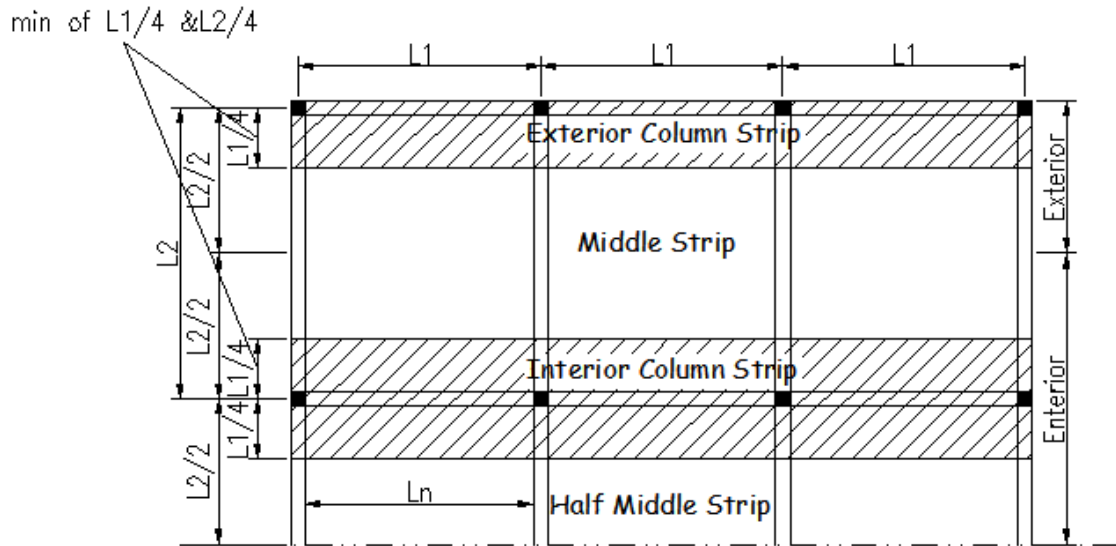
For the given data, design strip 1-2-3-4 of the two way slab for flexure.

Data:

Columns are 30cm X 30cm, Equivalent partitions load=250 Kg/m², Live Load = 400Kg/m²,
 $f'_c = 280 \text{ kg/cm}^2$, $f_y = 4200 \text{ Kg/cm}^2$, slab thickness = 16cm



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Solution:

Thickness is given 16cm, no need to be checked.

1- Calculate total factored load W_u "t/m²":

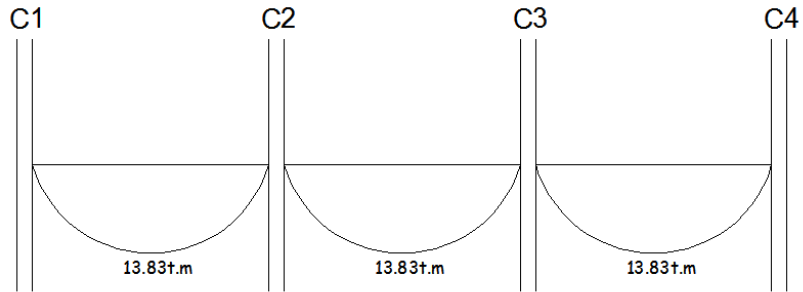
$$W_u = 1.4 \times (0.16 \times 2.5 + 0.25) + 1.7 \times (0.4) = 1.59 \text{ t/m}^2$$

2- Determine The Total Factored Static Moment (M_o) :

$$M_o = \frac{W_u \times \text{strip width} \times l_n^2}{8} = \frac{1.59 \times 3.15 \times 4.7^2}{8} = 13.83 \text{ t.m}$$

DESIGN OF REINFORCED CONCRETE STRUCTURES II
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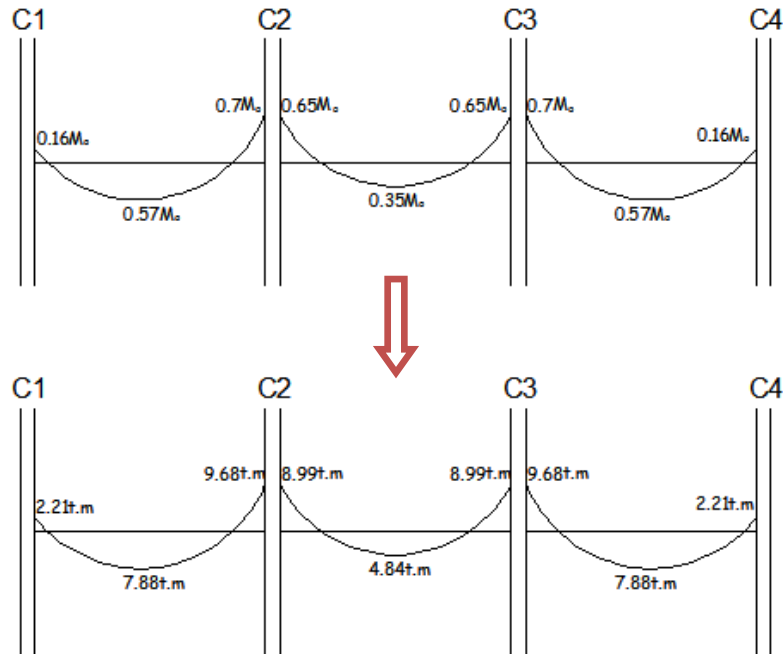
Total Factored Static Moment



3- Distribute M_o into +ve and -ve moments :

The total factored static moment was distributed according to Table "8.9" in your text book as shown in the following Figure.

Positive and Negative Moments



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4- Moments on the column Strip :

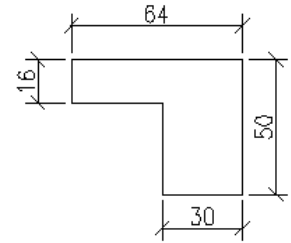
Evaluate the constant α and β

- Evaluation of α :

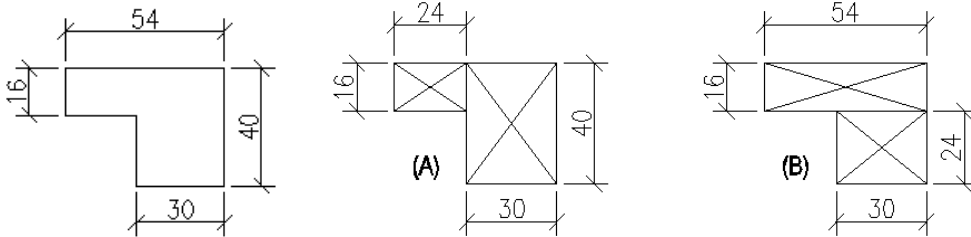
$$\alpha = \left(\frac{b}{\text{strip width}}\right) \left(\frac{a}{h}\right)^3 f. \text{ "For beam in direction of strip"}$$

For $a/h=50/16=3.125$ and $b/h=30/16=1.875$, $f=1.4$ (Fig. 8.14)

$$\alpha = \left(\frac{0.3}{3.15}\right) (3.125)^3 \times 1.4 = 4.07$$



- Evaluation of β :



$$\beta_t = \frac{C}{2I_s}. \text{ " For edge beam perpendicular to direction of strip"}$$

$$C_A = \left(1 - 0.63 \times \frac{16}{24}\right) \left(\frac{16^3 \times 24}{3}\right) + \left(1 - 0.63 \times \frac{30}{40}\right) \left(\frac{30^3 \times 40}{3}\right) = 208,909.55 \text{ cm}^4$$

$$C_B = \left(1 - 0.63 \times \frac{16}{54}\right) \left(\frac{16^3 \times 54}{3}\right) + \left(1 - 0.63 \times \frac{24}{30}\right) \left(\frac{24^3 \times 30}{3}\right) = 128,532.48 \text{ cm}^4$$

$$C = \text{Max}(C_A \text{ or } C_B) = 207393.8$$

$$\beta_t = \frac{208,909.55}{\frac{1}{12} \times 315 \times 16^3 \times 2} = 0.97$$

After α and β_t are calculated, factors for the moment of column strip can be found

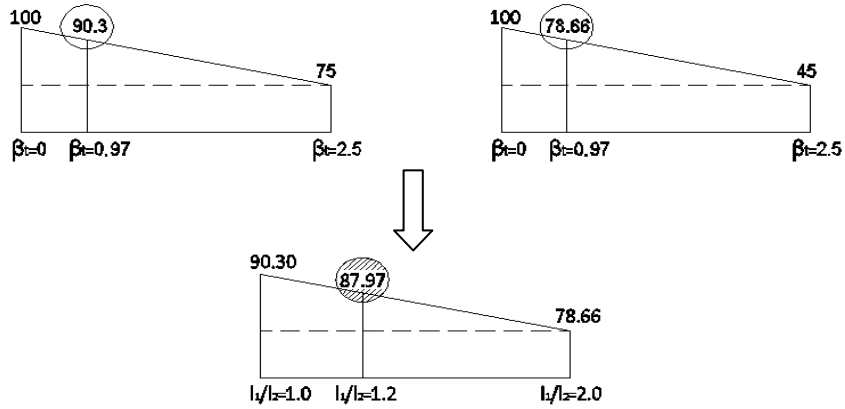
from Table 8.10, page 343

$$\alpha \frac{l_2}{l_1} = 4.07 \times \frac{6}{5} = 4.88 > 1, \quad \beta_t \text{ between } 0 \text{ and } 2.5, \quad \frac{l_2}{l_1} = 1.2$$

- Ve exterior moment Factor :

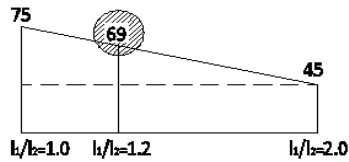
	$l_1/l_2=1$	$l_1/l_2=1.2$	$l_1/l_2=2$
$\beta_t = 0$	100	100	100
$\beta_t = 0.97$	0.903	0.8797	0.7866
$\beta_t = 2.5$	75	69	45

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- +Ve interior moment Factor :

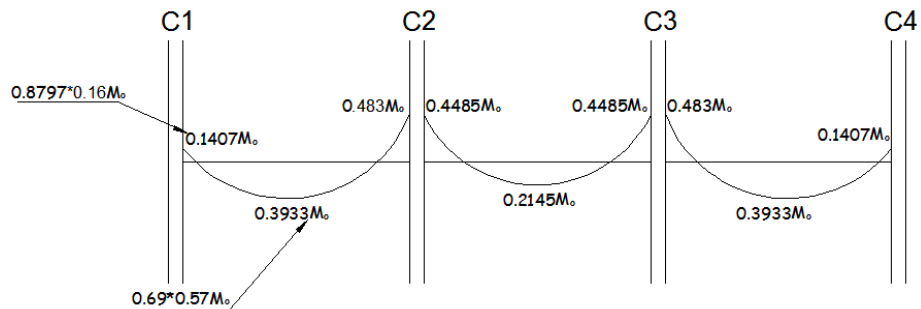
$l_1/l_2=1$	$l_1/l_2=1.2$	$l_1/l_2=2$
75	69	45



- -Ve interior moment Factor :

$l_1/l_2=1$	$l_1/l_2=1.2$	$l_1/l_2=2$
75	69	45

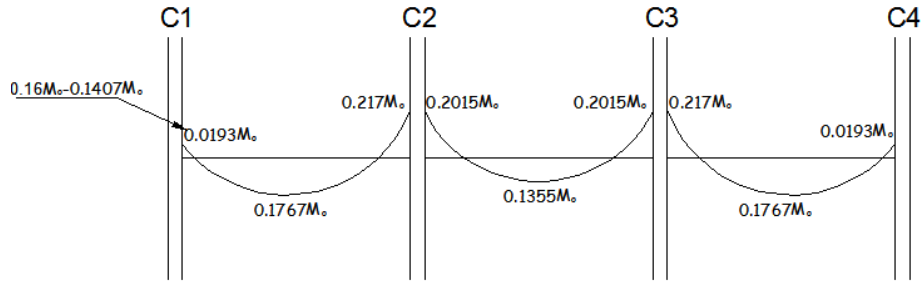
Column Strip Moments



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5- Moments on the Middle Strip: "The remain moment":

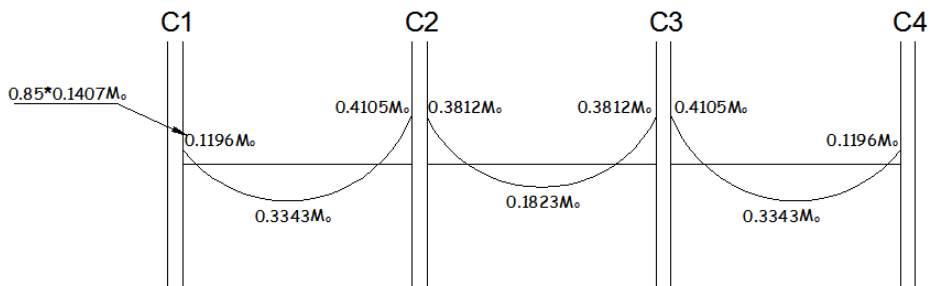
Middle Strip Moments



6- Moment On Beam :

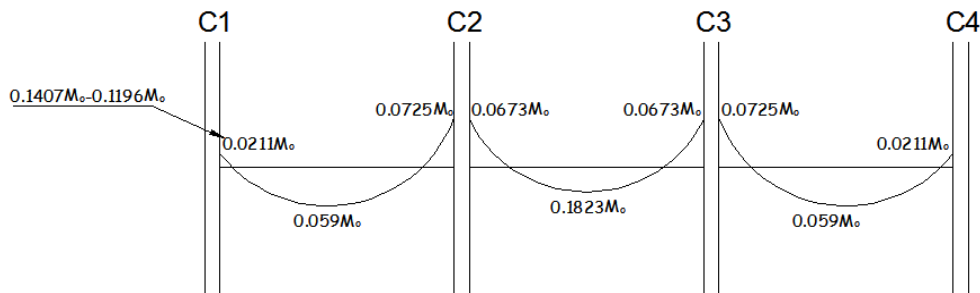
As $\alpha \frac{l_2}{l_1} > 1$ Beam will resist 85% of the column strip moment.

Beam Moments



7- Moment On Remained Slab :

Remain Slab Moments



DESIGN OF REINFORCED CONCRETE STRUCTURES II
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Notes:

- For each value of moment, ρ can be calculated, then A_{st} .
- **Widths to used for design and ρ calculations are :**
For the remained slab of column strip: $b = 1.25 - 0.3 = 0.95\text{m}$
For half middle strip: $b = 3.15 - 1.25 = 1.9\text{m}$
Beam = 0.3m
- Beam should be designed for shear, according to specifications of code **ACI 318"13.6.8"**, and reported in page 344 of your text book.

By:

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Eng. Nour Nagi Al-Hindi